PLANCKS
London 2020
0th online edition

This PLANCKS competition contains 15 pages (including this cover page) and 9 problems. Please work on paper and put both your team's name and question number on the top of every page.

You are required to show your work on each problem on this exam. The following rules apply:

- The contest consists of **9 problems, each worth 10 points**. Subdivisions may be indicated in the problems.
- **Organize your work**, in a reasonably neat and coherent way. All problems must be done on paper, in English, and scans uploaded before the end of the submission period into your team folder, which can be found at www.plancks.uk/exam. Files must be named appropriately, with your team name and question number and page number on every page. **Please upload the answer to each question as a separate file**.
- When a problem is unclear, a participant can ask, via the zoom call, for a clarification. If the response is relevant to all teams, the jury will provide this information to the other teams.
- The use of hardware (including phones, tablets, etc.) is not approved, except of scientific, non programmable calculators, watches and medical equipment. The use of computers should be limited to receiving/uploading the questions and communicating with your team and the jury. **Internet resources should not be used to answer questions**.
- The organisation has the right to disqualify teams at any point for misbehaviour or breaking the rules.
- In situations to which no rule applies, the organisation decides.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total:	90	

1 A Snow Plough Problem

(a) (10 points) On a snowy winter's morning a snow plough leaves the yard at 06.00 am precisely and makes its way along a long straight road. The driver notices that in the second hour she has travelled half the distance that she did in the first hour. **At what time did it start snowing?**

State a small set of reasonable assumptions which lead to the following set of variables and parameters:

Suitable variables and parameters

- Time of travel t, with $t = 0$ at 06.00
- Distance travelled by plough $x(t)$
- Height of snow $h(t)$
- Rate of snowfall per unit area R_S
- Rate of snow removal by plough R_P
- Width of snow plough blade w
- Time snow is falling before plough starts T

All measured in SI units

Obtain:

i. a relationship between w, h, R_P, Δx and Δt, where the plough moves a short distance, Δx, in a time interval Δt

ii. a relationship between h, R_S, t and T

Professor Alan Davies - University of Hertfordshire

2 Dotty Dinosaur

(a) (10 points) In the toddler game 'Dotty Dinosaur', there is a fair dice with 6 colours. After rolling a colour, you collect a dot for your dinosaur of that colour – if you have already got a dot of that colour you don't do anything. You 'win' and the game ends when you have collected all six dots, one of each colour.

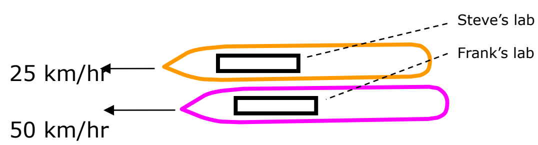
Calculate the mean number of throws of the dice needed to 'win' a game of dotty dinosaur.

Dr Sam Carr – University of Kent

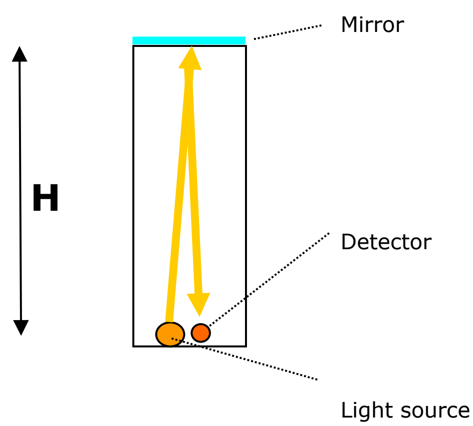
3 Einstein's Story

In this story some parts are missing! You are asked to fill in the blanks labelled shown in red. These numbered blancks could be an equation, expression or a sketch. No tricks! Very straightforward!

(a) (10 points) Suppose that two experimenters, Frank and Steve, are carrying out identical measurements using light on board two liners that are crossing the Atlantic in calm weather. Frank has his laboratory on the deck of one liner and Steve has his laboratory on the deck of the other liner. The liner with Frank on board is travelling at 50 km/hr and the liner with Steve on board is travelling at 25 km/hr in the same direction. Both laboratories have large windows so Frank can look into Steve's lab from his own lab and vice versa for the period that the liners are side by side, with the faster overtaking the slower:



Both experimenters each have an identical piece of equipment:



A light source sends a pulse of light vertically upwards to be then reflected vertically downwards by a mirror. A highly accurate detector measures the time for the up-down journey of the pulse.

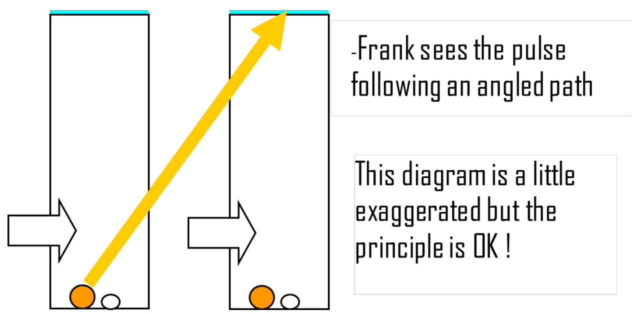
Both Frank and Steve carry out the same experiment each in their own lab and calculate the speed of light **C** by measuring the time of flight **T** of the pulse:

$$\boxed{1} \quad (1)$$

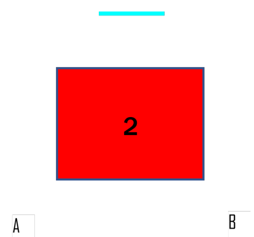
They talk by phone and find that each has found that their two results for the speed of light **C** are exactly the same. They are not surprised by this because, although one lab was moving at 25 km/hr relative to the other, neither lab was accelerating. Specifically, these experimenters

are remembering from their university physics lectures that:
The 'laws' of physics are the same for all experimenters in non-accelerating motion.

They now carry out a second experiment by adjusting each detector so that it can measure the time of flight of the pulse in the *other* laboratory. Frank's liner is overtaking Steve's and Frank sees Steve and his lab and his equipment moving to his (Frank's) right:



Frank sees the pulse of light in Steve's equipment start its upward journey and sees Steve's mirror move to his right during the time taken for the pulse to reach the mirror. After reflection, Frank sees the pulse following an angled path downwards as shown in this diagram:



Steve sees Frank and his lab moving to his right as Frank's liner overtakes. When they compare notes, both Steve and Frank find that they have observed the same angled-path phenomenon. They now do some analysis and write down the following:
Let $\boldsymbol{T'}$ *be the time taken for the pulse to travel from the source and back to the detector as measured by the experimenter on board the other liner. During this time the relative movement of one liner relative to the other* $\boldsymbol{AB}$ *is* 

where $\boldsymbol{V}$ *is the speed of one liner relative to the other. The distance* $\boldsymbol{D}$ *travelled by the pulse according to the experimenter in the other liner is:*

Therefore, the speed of light $\boldsymbol{C'}$ *according to the experimenter in the other liner, is* 

so:

(2)

Steve and Frank ponder on this result. They have recently heard that a guy called Einstein has been doing some work concerning the velocity of light but at this moment in time they are unaware of Einstein's conclusions. So Steve and Frank do the 'obvious' thing and agree that the time for the pulse of light to travel up and down vertically in one lab 'has to be exactly the same' as the pulse to travel up and down along the angled path in the other. That is, they assume that $\mathbf{T} = \mathbf{T}'$ and combine Equations 1 and 2 to obtain:

7 (3)

This result they find most interesting, for it suggests that when an experimenter measures the speed of light $\mathbf{C}'$ in a laboratory that is moving relative to him, the measured speed is greater than that which he would measure in his own lab. The greater the difference in relative speed between the laboratories, the greater the difference between the two speeds of light.

Steve and Frank agree that their result is in fact a Earth-shaker and they get down to preparing a paper for publication.

They then manage to get hold of a copy of Einstein's 1905 paper and are disturbed by it. According to Einstein, every measurement of the speed of light (in a vacuum) – whatever the speed of the experimenter relative to the source of the light - will yield the same result. They can see that, if Einstein had been helping them with their experiments on board the two liners, then he would have argued that the speed of light as measured by Steve in his own lab and by Frank looking in on Steve's equipment in Steve's moving lab would be the same. The same result would be found by Steve looking in on Frank's experiment. Einstein would have argued that $C =$ 8 .

Our two brave experimenters now make the assumption 9 $=$ and combine Equations 1 and 2 and, after a bit of algebra, they obtain:

10 (4)

Again, they ponder on what this means. There is no escaping from the fact that Equation 4 is telling them that, according to Frank looking in on Steve's moving lab, Steve's time $\mathbf{T}'$ was passing more slowly than his (Frank's) time $\mathbf{T}$ because the denominator of 4 will always be less than 1. And Steve would draw exactly the same conclusion about the rate of passing of Frank's time in Frank's moving lab. That the rate of passage of time is not the same for everyone is a more odd-ball result than that of the speed of light increasing when one looks in on a light-speed measurement experiment that is on the move!

Steve and Frank are faced with a choice. If they assume $\mathbf{T{=}T}'$ then they obtain Equation 3. If they assume 9 $=$ then they obtain Equation 4. Which one is right? They put the publishing of their epic paper on hold!

Dr John Davis - University of Hertfordshire

4 Time of Flight

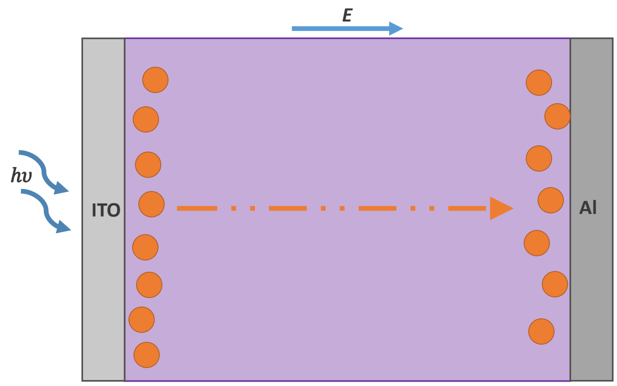


Figure 1: A sample displaying transient time of flight. With $h\nu$ denoting the incident photon and E the applied bias. With the electrodes being Indium Tin Oxide (ITO) and Aluminium.

Transient Time of Flight, ToF, is a widely used tool for measuring mobility, μ. The principle is that by measuring transient photocurrents, one can find the time it takes for a packet of charge carriers to reach across the bulk of the testing material when driven by an applied bias voltage. The electron-hole pairs are generated by excitation, typically using a single pulsed laser. These are generated near the incident electrode and will travel together across the sample being measured as a charge packet in the direction of the appropriate voltage bias. Once at the far electrode, the charge carriers can be extracted and the transit time, t_{tr}, measured. This process is depicted in figure 1.

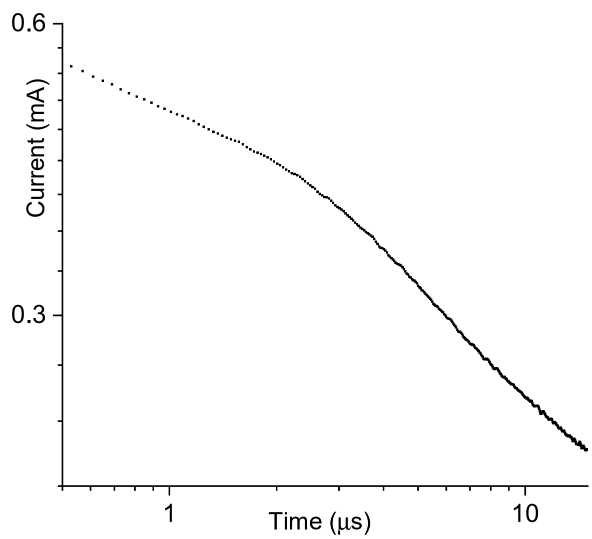


Figure 2: Transient photocurrents on a log-log time graph

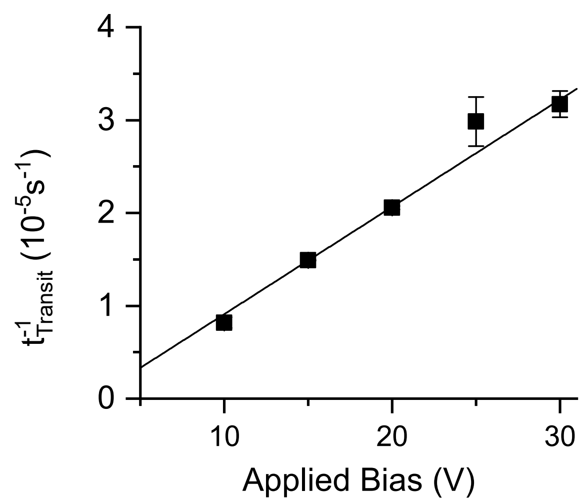


Figure 3: Inverse transit times scaling with applied bias.

(a) (2 points) Figure 2 shows a typical log-log plot of photocurrent against time. Deduce from this graph where the transit time will be and explain why you have chosen this position. You do not need to give an actual value of the transit time.

To find the mobility, one uses the relation:

$$\mu = \frac{d^2}{V t_{tr}}$$

Where V is voltage and d denotes the device thickness, typically $1.33 \pm 0.07 \mu m$. One can plot this linear relationship of inverse transit time against a applied voltage bias, as can be shown in figure 3.

(b) (2 points) From figure 3, calculate the charge carrier mobility of this sample. Include an appropriate error in your result.

(c) (2 points) We have not yet discussed what kind of charge carriers we are measuring. Using the information already provided, what charge particle have we measured mobility for? What happens to the opposite charge?

(d) (1 point) The charge carriers can get trapped while moving through the bulk of the sample on the way to the electrode. Give a example of one such kind of trap.

(e) (2 points) How will the charge particles being trapped affect the charge packet and how will figure 2 change?

(f) (1 point) What will figure 2 look like if we didn't apply a bias voltage? Explain why.

Dr James Kneller - Queen Mary, University of London

5 Phonon-assisted charge transport in 2D

Consider a two-dimensional monatomic crystal with a square lattice. In this problem we choose two neighbouring sites A and B, away from the boundaries of the material, and calculate the resistance between them.

(a) (5 points) In this material, charges q hop between the neighbouring sites to lower their energy, while simultaneously ejecting phonons (bosonic quanta of lattice vibrations). Here, we focus on the system constituted only by the neighbouring sites A and B, coupled to the phonon reservoir. The dynamics is modelled by the following Hamiltonian:

$$H = \frac{qU}{2}(|A\rangle\langle A| - |B\rangle\langle B|) + \hbar v \sum_{\underline{k}} |\vec{k}| n_{\underline{k}} + g \sum_{\underline{k}} c^{\dagger}_{\underline{k}} |B\rangle\langle A| + c_{\underline{k}} |A\rangle\langle B| \tag{5}$$

Where U is a voltage difference between the sites A and B, v is a phonon group velocity, g is a strength of a phonon-jump coupling, $c^{\dagger}_{\underline{k}}$ is an operator creating a phonon with a wavevector $\underline{k}$, and $n_{\underline{k}}$ is a corresponding phonon number operator.

Assuming that the charge q is initially on the site A, and that the phonon reservoir is empty, **estimate the resistance R between the sites**. The area of the material is S.

(b) (3 points) Of course, the charge can hop to each of the neighbouring sites. Here we apply a semi-classical approximation to account for this effect. Consider all the edges between the lattice sites to be resistors with a resistance R. **Calculate the effective resistance between the neighbouring sites** A **and** B assuming that the area of the material is much greater than the area of a unit cell, and that the current vanishes at the boundaries of the material.

(c) (2 points) How would this result change if sites were distributed on a honeycomb, but phonon properties remained unchanged?

In the last two parts of the question you can take R as given or your answer to part a.

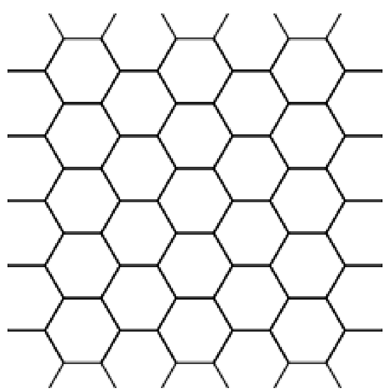


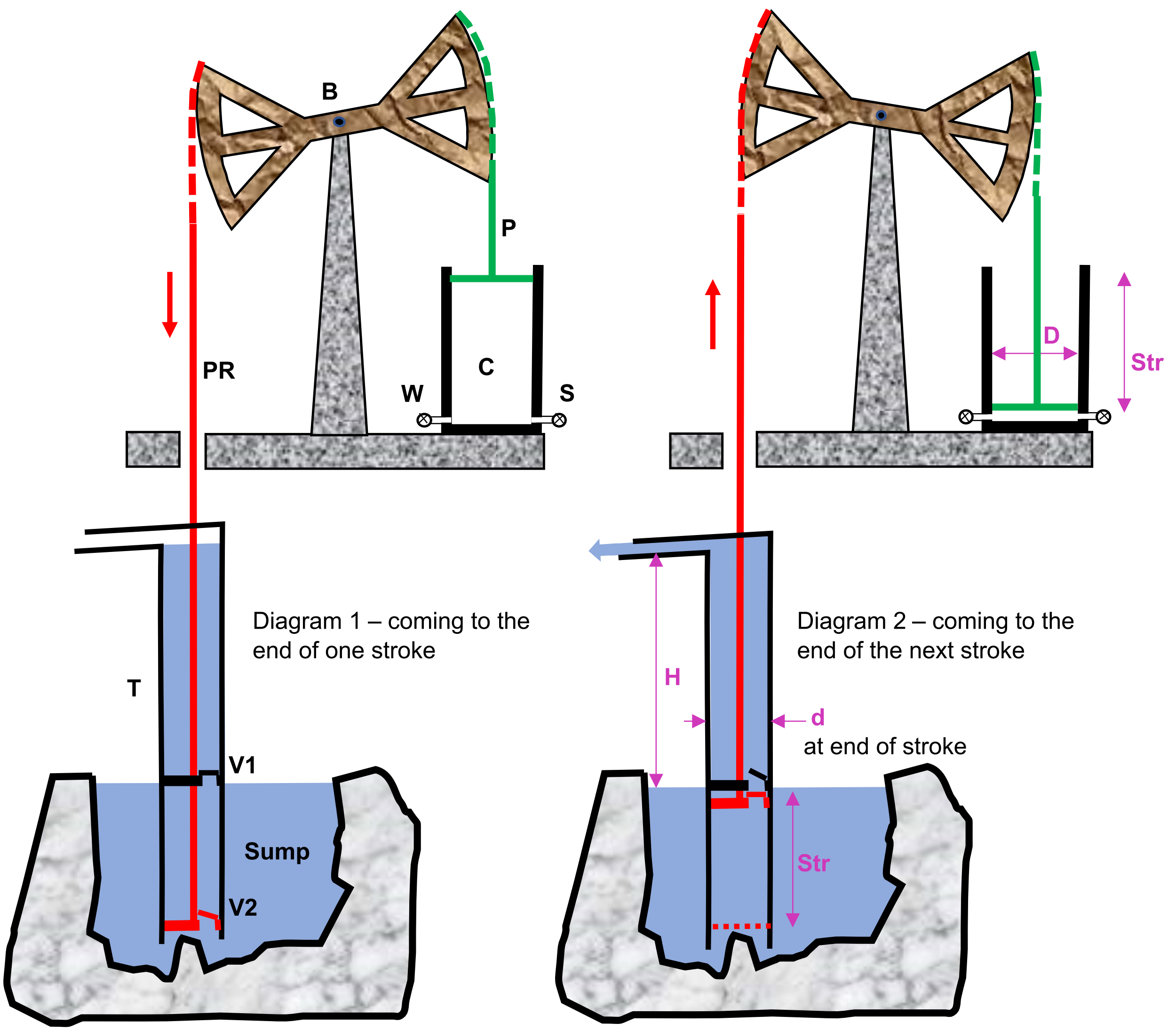
Figure 4: Honeycomb

Leon Zaporski, Quantum Optical Materials and Systems - U of Cambridge

6 Newcomen's Beam Engine

The Industrial Revolution in Britain started when 'steam engines' began to supplement and then replace wind, water and *horse power*. The first of such engines was built by Thomas Newcomen in 1712 and was used to drain water from a coal mine that was about 50m deep. By the time of Newcomen's death in 1739 about a hundred of his engines were in operation in Britain. These were all beam engines. Their design was much improved by James Watt from 1760 onwards - with the oscillatory motion of beam engines being adapted in about 1780 to produce the more useful, rotary motion. All that will be now said is that during the next hundred and fifty years a huge range of steam engine variants emerged. There are several examples of Newcomen's Engine in existence, including one at the Science Museum, London.

Newcomen's original engine was massive, being about the size of a large house. Here is a hypothetical engine based roughly on Newcomen's design. It has the same operating principle of his engine but with quite different proportions appropriate for present purposes. The boiler supplying steam is not shown.



The large (wooden) beam **B** pivots about its mid-point. Its rocking motion raises and lowers a piston rod **P** and piston (green) in the 'steam' cylinder **C** and a very long and heavy pump rod **PR** (red) that descends down the mine shaft. This latter rod moves within a fixed, cylindrical tube **T** that dips into the sump into which the mine water collects. **V1** is a 'flap' valve fixed within **T**. **V2** is also a 'flap' valve that is part of the piston attached to the end of the pumping rod. The weight of the (red) pumping rod **PR** and piston *is* intentionally greater than the weight of the (green) piston rod **P** and piston.

The original engine did not run by itself and required an operator to open and shut valves in sequence. Diagram 1 shows the engine just before the end of the pumping rod's downward motion which ends by the piston 'thumping' against a 'stop'. Several cycles of engine operation have already taken place in order to prime the pump. The steam valve **S** has been open allowing steam at atmospheric pressure to enter cylinder **C**. The greater weight of the pumping rod has caused it to descend with valve **V2** open and, as the piston at the end of the rod moves down into the sump, sump water enters the lower part of the cylindrical tube **T**. Valve **V1** has remained shut as the pumping rod has moved downwards.

The operator now shuts off the steam supply valve **S** and for a very short time opens the valve **W** to introduce a fine spray of cold water into **C**. This condenses the steam and causes a vacuum to develop within **C**. Atmospheric pressure acting on the (green) piston begins to push it downwards and to begin to raise the pumping rod **PR**. Valve **V1** now opens and valve **V2** shuts. Diagram 2 shows the end of this stroke with the (green) piston about to 'bottom' and stop. During the upward, 'working' stroke of the engine water inside **T** is raised so as to flow out of the mine at ground level. The operator now opens the steam valve **S** to 'kill' the vacuum in **C** and to start the next cycle of operation.

(a) (10 points) **You are asked to calculate the volume of water raised up and removed from the sump per cycle of engine operation using the following information.**

A cycle is simply a downward and an upward stroke of **PR** or, equivalently, an upward and a downward stroke of **P**, with both strokes being of the same length. Not surprisingly, your answer will depend upon the assumptions that you make so it is important for you to make a list of these.

- **Excess mass of pumping rod PR over rod P, ExMass = 1200 kg**
- **Stroke, Str = 2 m**
- **Diameter, D = 0.8 m**
- **Diameter, d = 0.3 m**
- **Atmospheric pressure, AtmP = 0.1 MPa**
- **Height, H = 50 m**

Dr John Davis - University of Hertfordshire

7 1888

The following set of questions are taken from a first year physics and maths exam of the University of Sydney for the year 1888. We will accept answers both in line of the understanding of the time and our current thinking. You will need to answer all parts of the questions to get the point.

(a) (1 point) Explain why the moon does not fall on to the earth.

(b) (1 point) What was the famous experiment of Archimedes relating to the volume of water displaced from a full vessel by a body immersed in it? How is the principle applied in finding specific gravity?

(c) (1 point) Explain as fully as possible the effects of—
   i. Raising the temperature of a volume of air enclosed in a vessel.
   ii. Increasing the pressure; while the temperature is kept constant.

(d) (1 point) Explain exactly how you would set about making a Barometer.
What does a barometer measure ?

(e) (1 point) Give some explanation of the phenomenon of lightning.
What is the supposed use of lightning conductors?

(f) (1 point) Draw a diagram explaining the working parts and mode of action of the ordinary steam engine.

(g) (1 point) What is meant by the term electrolysis? Illustrate your answer by some simple example, and note the variations in the state of the energy of the system during the process.

(h) (1 point) What is a magnet? How would you make one?

(i) (1 point) Draw a diagram and explain the action of an ordinary Photographic Camera.
Why do we fail, as a rule, in attempting to take a photograph by gaslight?

(j) (1 point) Shew that:

$$\int_0^\infty e^{-a^2x^2} \cos(2bx)\, \mathrm{d}x = \frac{\sqrt{\pi}}{2a} e^{-\frac{b^2}{a^2}}$$

.

UNIVERSITY OF SYDNEY, 1888
Answers provided by Anthony Quinlan - University of Durham

8 Dominoes and Pendulums

(a) (6 points) Consider Figure 5 (a) which shows a uniform two dimensional domino of height $2l$ and width $2d$ with $d < l$. The domino is placed such that the bottom right bottom corner is at the origin and it follows that the centre of mass of the domino is at the position $(-d; -l)$. For $d < l$ the domino has lower energy when lying on its side, so that the centre of mass is at the position $(l; d)$, see Figure 5 (b). Why doesn't the domino achieve its minimum energy by spontaneously falling on its side?

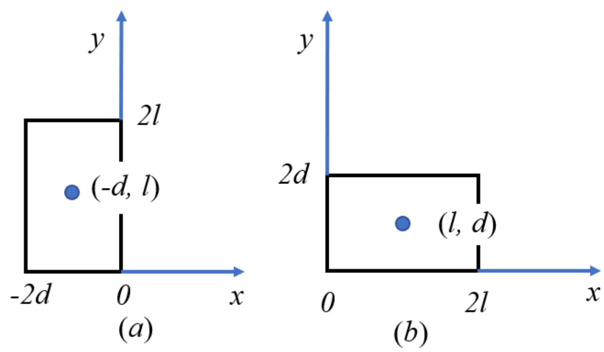


Figure 5: Domino in (a) high energy and (b) low energy state.

(b) (4 points) Figure 6 shows a simple pendulum, where a bob of mass m is suspended by a light, inextensible and taut rod. The bob is displaced by an angle θ and moves under the in influence of gravity with the acceleration due to gravity is g.

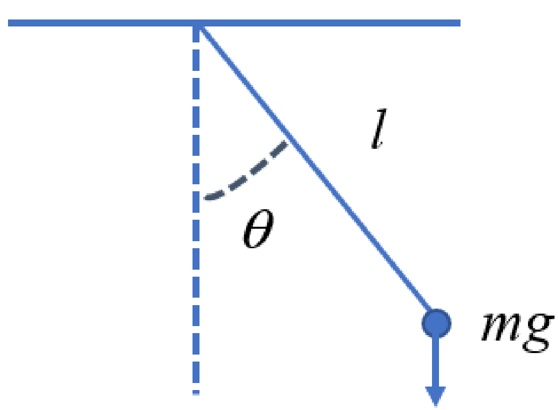


Figure 6: The simple pendulum.

The equation of motion when the bob is displaced by the angle θ is easily derived and found to be:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l} sin\theta = 0 \tag{6}$$

For small angles the approximation $sin\theta \approx \theta$ is valid and the equation of motion becomes:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \tag{7}$$

and the solution for the ensuing motion when the initial conditions are $\theta_0 = \theta + 0, \frac{d\theta}{dt(0)} = 0$, i.e. the bob starts from rest from the angle θ_0 is:

$$\theta(t) = \theta_0 cos\sqrt{\frac{g}{l}}t\theta = 0 \tag{8}$$

One implication of this is that the period of the motion, T, is given as $T = 2\pi\sqrt{\frac{l}{g}}$ and is independent of the amplitude (as long as the amplitude is small enough such that the approximation of $sin\theta \approx \theta$ remains valid during the motion).

Present a simple argument based on continuity that suggests that the period increases as the initial amplitude increases such that the approximation of small angle is not justified.

James Mc Tavish - Liverpool (Retired)

9 Buried Room

(a) (10 points) Imagine that you could build a large room (10m x 10m x 10m) at the gravitational centre of the earth. Would you float in the room, or stand on one of the walls?

Record the principles, assumptions and considerations made in arriving at your conclusion.

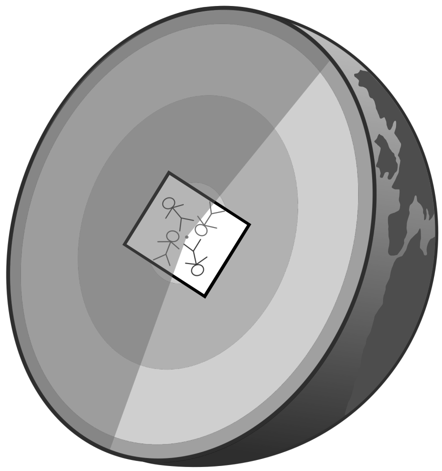


Figure 7: Sketch not to scale, for illustrative purposes only and not indicative of the answer.

Andrew Inkersole - IOP member